LESSON 7

Representing and Interpreting Proportional Relationships

UNDERSTAND A proportional relationship can be represented in different ways.

A right whale eats an average of 2 tons of plankton every day. The relationship between the number of days and the number of tons of plankton eaten can be expressed in a table.

Number of Days, x	Amount of Food Eaten (in tons), y
1	2
2	4
3	6
4	8
5	10

The ratio of the y-value to the x-value will always be 2:1. Similarly, the y-value always changes by the same amount $(+2)$ when the x-value changes by the same amount $(+1)$. Therefore, the relationship is proportional.

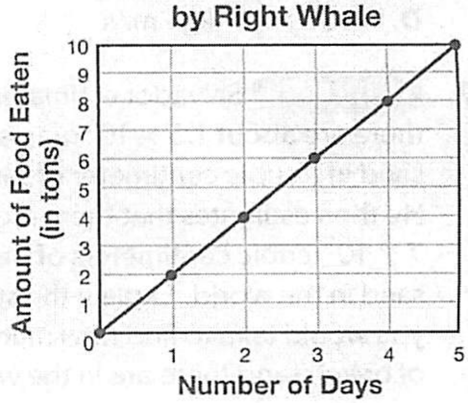
Another way to represent the proportional relationship in the table is with an equation. All proportional relationships have the form $y = kx$, where k is any nonzero number.

$$y = 2x$$

You can also represent a proportional relationship with a graph, as shown at the right. Using the x- and y-values from the table as ordered pairs or using the equation, you can plot the points on a coordinate plane and draw a straight line through the points. The graph of a proportional relationship always passes through point (0, 0), the **origin**.

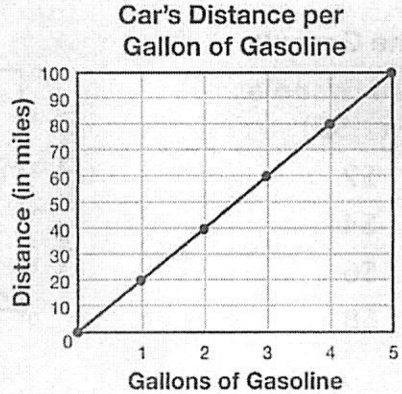
The **slope** of a line is a ratio that compares the change in y-coordinates (the rise) of a graph to the change in x-coordinates (the run). In the line graphed at the right, the y-coordinates increase by 2 as the x-coordinates increase by 1, so the slope is $\frac{2}{1}$, or 2.

Amount of Food Eaten by Right Whale



Connect

The following graph represents the distance a car can travel based on the number of gallons of gas. The relationship is proportional.



Find the miles per gallon, or **unit rate**, for the car.

1 Find the graph's rate of change to determine the unit rate.

The **rate of change** can be found by determining the slope. Choose any two points, (x_1, y_1) and (x_2, y_2), on the graph and write the ratio.

$$\frac{\text{change in } y\text{-values}}{\text{change in } x\text{-values}} = \frac{y_2 - y_1}{x_2 - x_1}$$

2 Substitute (2, 40) for (x_1, y_1) and (4, 80) for (x_2, y_2).

$$\frac{\text{change in } y\text{-values}}{\text{change in } x\text{-values}} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{80 - 40}{4 - 2} = \frac{40}{2}$$

3 Simplify the fraction so that the denominator is 1. The resulting fraction will be the unit rate.

$$\frac{40}{2} = \frac{20}{1}$$

The unit rate represents the number of miles per 1 gallon.

▶ The unit rate is $\frac{20 \text{ miles}}{1 \text{ gallon}}$, or 20 mi/gal.

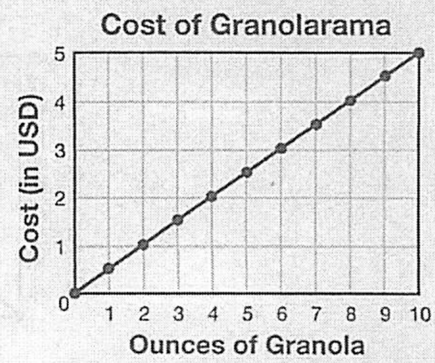
TRY

Compute the unit rate using two different points from the line. What do you notice?

EXAMPLE A A grocery store sells two varieties of trail mix: Wholesome Granola and Granolarama. Look at the table and graph.

Cost of Wholesome Granola

Ounces of Granola	Cost of Granola (in USD)
5	$2
10	$4
15	$6
20	$8



Which granola is the better buy?

1 Compare the unit rates.

The unit rate is the price of granola for 1 ounce. The better buy will be the granola with the lower unit rate.

2 Determine the unit rate of Wholesome Granola from the table.

To find the unit cost, divide the cost by the number of ounces.

$2 ÷ 5 oz = $0.40 per ounce

$4 ÷ 10 oz = $0.40 per ounce

$6 ÷ 15 oz = $0.40 per ounce

$8 ÷ 20 oz = $0.40 per ounce

The unit rate of Wholesome Granola is $0.40 per ounce.

3 Determine the unit rate of Granolarama from the graph.

Compare the change in y-coordinates (the rise) to the change in x-coordinates (the run). The line passes through (0, 0) and (10, 5).

$$\frac{\text{change in } y\text{-values}}{\text{change in } x\text{-values}} = \frac{5-0}{10-0} = \frac{5}{10} = 0.5$$

The unit rate of Granolarama is $0.50 per ounce.

4 Compare the unit rates.

$0.40 < 0.50$

▶ Wholesome Granola costs less than Granolarama per ounce. Wholesome Granola is the better buy.

MODEL

Write an equation to represent the information for each kind of granola.

EXAMPLE B Gus's Gaseteria sells gasoline using the following equation, where C is the total cost and g is the number of gallons of gasoline.

$C = 3.35g$

Sally's Station uses the following table to determine the total cost for buying different numbers of gallons of gasoline.

Cost of Gasoline at Sally's Station

Gallons of Gasoline	Cost of Gasoline (in USD)
2	$7
4	$14
6	$21
8	$28
10	$35

Which gas station's gasoline is the better buy?

1 Determine the slope from the Gus's Gaseteria equation.

An equation in the form "$y =$ " shows the slope as the coefficient of the x-value. It does not matter if different letters are used for the variables.

$C = 3.35g$

The slope of Gus's Gaseteria's equation is 3.35, so Gus's Gaseteria charges $3.35 per gallon.

2 Determine the slope from the Sally's Station table.

Use the first column of the table for the x-values and the second column for the y-values. For example, two ordered pairs would be (4, 14) and (8, 28).

$$\frac{\text{change in } y\text{-values}}{\text{change in } x\text{-values}} = \frac{28 - 14}{8 - 4} = \frac{14}{4} = 3.50$$

The slope from Sally's Station's table is 3.50, so Sally's Station charges $3.50 per gallon.

3 Compare the slopes.

$3.35 < 3.50$

▶ Gus's Gaseteria charges less than Sally's Station for 1 gallon of gasoline. Gasoline from Gus's Gaseteria is the better buy.

TRY

Terry charges $50 for 4 hours of tutoring. If this relationship between hours and charges were graphed, what would be the slope of the line?

Practice

Find each unit rate.

1. A bakery sells muffins using the equation $C = 1.85m$, where C is the cost of the muffins and m is the number of muffins.

   Unit rate = ____________ per muffin

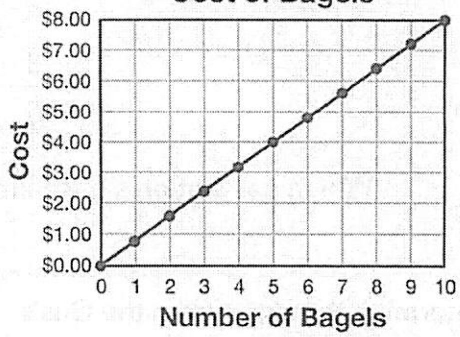
> REMEMBER An equation in the form "$y =$ " shows the unit rate in the coefficient of x.

2. **Cost of Croissants**

Number of Croissants	Cost
2	$3.50
4	$7.00
6	$10.50
8	$14.00

Unit rate = ____________ per croissant

3. **Cost of Bagels**



Unit rate = ____________ per bagel

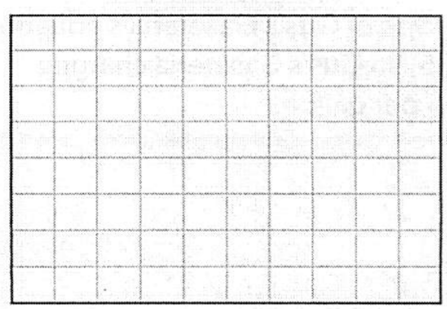
Graph the proportional relationship. Then find the unit rate.

4. A grocer uses the following table to show the cost of different numbers of lemons.

Cost of Lemons

Number of Lemons	Cost (in USD)
3	$1
6	$2
9	$3
12	$4

Cost of Lemons



Unit rate = ____________ per lemon

Find each slope.

5. A farmer charges for his coffee beans using the equation $C = 3.95p$, where C is the cost of the coffee beans and p is the number of pounds of coffee beans.

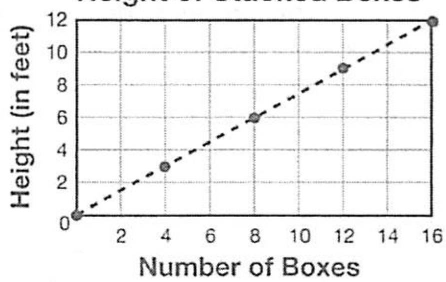
   Slope = ____________

6. **Cost of Scarves**

Number of Scarves	Cost (in USD)
4	$22.00
8	$44.00
12	$66.00
16	$88.00

   Slope = ____________

7. **Height of Stacked Boxes**



   Slope = ____________

Solve.

8. COMPARE The maximum distance traveled by the space shuttle can be determined using the equation $d = 4.8s$, where d is the distance, in miles, and s is the number of seconds. The table below shows the distance traveled by the Apollo 10 astronauts returning from the moon.

Distance Traveled by Apollo 10

Number of Seconds	Distance Traveled (in miles)
5	23.5
10	47
15	70.5
20	94

Compare the slopes to determine which craft—the space shuttle or Apollo 10—traveled at a greater speed, and explain the steps you took.

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