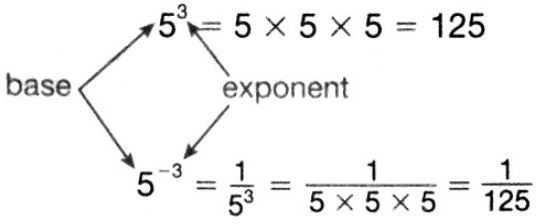
LESSON 3

Applying Properties of Exponents

UNDERSTAND In an exponential expression such as 4^7, the number 4 is called the **base**. The number 7 is called the **exponent**. It tells how many times the base is used as a factor. So, $4^7 = 4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4$.
If the exponent is negative, put the exponential expression in the denominator of a fraction with a numerator of 1. So, $4^{-7} = \frac{1}{4^7}$.



To multiply exponential expressions with the same bases, add the exponents.

$5^3 \times 5^2 = 5^{3+2} = 5^5$

Why this works:

$5^3 \times 5^2 = (5 \times 5 \times 5) \times (5 \times 5)$

$= (5 \times 5 \times 5 \times 5 \times 5) = 5^5$

To divide exponential expressions with the same bases, subtract the exponents.

$\frac{4^5}{4^3} = 4^{5-3} = 4^2$

Why this works:

$\frac{4^5}{4^3} = \frac{4 \times 4 \times \cancel{4} \times \cancel{4} \times \cancel{4}}{\cancel{4} \times \cancel{4} \times \cancel{4}}$

$= \frac{4 \times 4}{1} = 4^2$

To find the value of an exponential expression with an exponent, multiply the exponents.

$(3^2)^3 = 3^{2 \times 3} = 3^6$

Why this works:

$(3^2)^3 = 3^2 \times 3^2 \times 3^2$

$= (3 \times 3) \times (3 \times 3) \times (3 \times 3)$

$= (3 \times 3 \times 3 \times 3 \times 3 \times 3) = 3^6$

When a product has an exponent, the exponent applies to each factor.

$(9x)^3 = (9^3)(x^3) = 729x^3$

Why this works:

$(9x)^3 = 9x \cdot 9x \cdot 9x$

$= (9 \cdot 9 \cdot 9)(x \cdot x \cdot x)$

$= 9^3 \cdot x^3 = 729x^3$

When a quotient has an exponent, the exponent applies to each term.

$\left(\frac{3}{4}\right)^3 = \frac{3^3}{4^3} = \frac{27}{64}$

Why this works:

$\left(\frac{3}{4}\right)^3 = \frac{3}{4} \cdot \frac{3}{4} \cdot \frac{3}{4} = \frac{3^3}{4^3} = \frac{3 \times 3 \times 3}{4 \times 4 \times 4} = \frac{27}{64}$

Connect

Multiply: $7^6 \times 7^{-4}$

The expressions have the same base, 7, so add the exponents and simplify.

$$7^6 \times 7^{-4} = 7^{6+(-4)}$$
$$= 7^2$$
$$= 7 \times 7$$
$$= 49$$

▶ $7^6 \times 7^{-4} = 49$

Divide: $\frac{3^{-2}}{3^3}$

The expressions have the same base, 3, so subtract the exponents and simplify.

$$\frac{3^{-2}}{3^3} = 3^{-2-3}$$
$$= 3^{-5}$$
$$= \frac{1}{3^5}$$
$$= \frac{1}{3 \times 3 \times 3 \times 3 \times 3}$$
$$= \frac{1}{243}$$

▶ $\frac{3^{-2}}{3^3} = \frac{1}{243}$

Evaluate: $(2^{-3})^2$

The exponential expression has an exponent, so multiply the exponents. Then simplify.

$$(2^{-3})^2 = 2^{-3 \times 2}$$
$$= 2^{-6}$$
$$= \frac{1}{2^6}$$
$$= \frac{1}{2 \times 2 \times 2 \times 2 \times 2 \times 2}$$
$$= \frac{1}{64}$$

▶ $(2^{-3})^2 = \frac{1}{64}$

Evaluate: $\left(\frac{2}{5}\right)^{-3}$

Find the negative power of a quotient.

$$\left(\frac{2}{5}\right)^{-3} = \frac{2^{-3}}{5^{-3}}$$
$$= \frac{1}{2^3} \div \frac{1}{5^3}$$
$$= \frac{1}{2^3} \times \frac{5^3}{1}$$
$$= \frac{5^3}{2^3}$$
$$= \frac{5 \times 5 \times 5}{2 \times 2 \times 2}$$
$$= \frac{125}{8}$$

▶ $\left(\frac{2}{5}\right)^{-3} = \frac{125}{8} = 15\frac{5}{8}$

DISCUSS

How can you write $\frac{2^{-5}}{7^{-2}}$ as an exponential expression with positive exponents?

Practice

Evaluate each expression. Leave your answer in exponential form.

1. $3^4 \times 3^2$

_____

2. $9^7 \times 9^3$

_____

3. $6^3 \times 6^3$

_____

> **REMEMBER** You can multiply exponential expressions with like bases by adding the exponents.

4. $5^5 \times 5^4$

_____

5. $1^{11} \times 1^{-9}$

_____

6. $x^3 \times x^{-6}$

_____

7. $\frac{4^9}{4^7}$

_____

8. $\frac{2^{-10}}{2^4}$

_____

9. $\frac{z^{20}}{z^{10}}$

_____

Complete each sentence.

10. To evaluate $(4^3)^4$, I need to _____ the exponents.

11. To evaluate $\frac{10^6}{10^4}$, I need to _____ the exponents.

12. To evaluate $p^{-2} \times p^9$, I need to _____ the exponents.

Evaluate each expression. Write your answer in standard form.

13. $2^2 \times 2^3$

_____

14. $3^3 \times 3^2$

_____

15. $\frac{6^7}{6^5}$

_____

16. $\frac{9^9}{9^8}$

_____

17. $\left(\frac{1}{4}\right)^3$

_____

18. $\left(\frac{4}{9}\right)^2$

_____

Evaluate each expression. Write your answer in standard form. Justify each answer.

19. $5^{10} \times 5^{-7}$

20. $7^{-4} \times 7^{6}$

21. $\frac{2^{-5}}{2^{-8}}$

22. $\frac{10^{-11}}{10^{-10}}$

23. $1^{17} \times 1^{-8}$

24. $\left(\frac{2}{3}\right)^{-3}$

Choose the best answer.

25. Which expression is equal to $\frac{13^{-11}}{13^{-12}}$?

A. 13^{-23}

B. 13^{-1}

C. 13

D. 13^{132}

26. Which expression is equal to $(5^4)^{-3}$?

A. 5^{-12}

B. 5^{-7}

C. 5

D. 5^{12}

Solve.

27. **CHOOSE** Choose a rational approximation for π. Then evaluate $\frac{\pi^7}{\pi^5}$.

28. **RESTATE** In your own words, explain how you know that $(d^6)^2$ is equal to d^{12}.